Study of Natural Down-welling Sky Light with Imaging Spectropolarimeter

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ABSTRACT

A full sky imaging spectropolarimeter that measures spectrally resolved (~2.5 nm resolution) radiance and polarization (s_0 , s_1 , s_2 Stokes Elements) over approximately 2π sr between 400nm and 1000nm will be used to quantitatively characterize the spectral dependence of the polarization state of the sunlight scattered in the sky. The sensor is based on a scanning push broom hyperspectral imager configured with a continuously rotating polarizer (sequential measurement in time polarimeter). This study will help optimize sky polarimetry by offering information that can be used to select the best spectral band (or which spectra to reject) for a given application. Findings to be presented are sky maps of the angle of polarization and degree of polarization for different spectral bands, spectral dependency of degree (and angle) of polarization, and example data sets supporting each.

1. Introduction

Polarization induced by scattering from molecules within the atmosphere is well described using Rayleigh theory, a subset of Mie theory¹; Rayleigh theory applies when light interacts with an object very small compared to the incident wavelength of the light.² The primary molecules responsible for the scatter are dipoles (N_2 and O_2) but are much smaller³ than the wavelengths of visible light, so Rayleigh theory is a suitable approximation⁴. A second approximation made in this paper is that the light only scatters once before reaching the observer. The first scatter is much brighter than higher order scatters, but light can scatter many times before reaching the observer. The most apparent effects of this multiple scattering can be seen near the sun and directly opposite the sun (antisun) ⁴. These approximations also break down when sunlight is scattered by pollutants, clouds, or other atmospheric particles. However, in clear weather conditions single scatter Rayleigh theory proves to be a good baseline.

Motivation

This research will expand the quantitative understanding of scattered sunlight in the sky. Polaris Sensor Technologies has a vested interest in the nature of sky light because of the in-house developed navigation solution called SkyPASS (Sky Polarization Azimuth Sensing System). SkyPASS (Figure 1) uses the polarization pattern in the sky to determine heading in real time and without drift. The goal for this research is to optimize the performance of this sensor (current heading rms error is 4 milliradians). A deeper understanding of the sky's deviation from single-point Rayleigh scatter theory as well as its spectral dependencies can lead to design changes and improved algorithms to most effectively relate sky polarization to observer orientation.



Figure 1: SkyPASS

Polarization and Theory

As we shall see, the atmosphere (theoretically) has no impact on circular polarization and natural sunlight is unpolarized, so the focus on this paper will be linear polarization. It should also be noted that the electric field oscillates in a direction perpendicular to the direction of wave propagation; the direction of oscillation (E vector) can be quite different than the direction implied by the apparent angle between the E vector and a reference axis (angle of polarization or AoP). The attributes of the polarization state most desired are the AoP and degree of linear polarization (DoLP). Acquisition of these values requires the Stokes vector:

$$S = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} I_0 + I_{90} \\ I_0 - I_{90} \\ I_{45} - I_{135} \\ I_L - I_R \end{bmatrix}$$
(1)

In Eq. 1, I is the radiance collected by the sensor with varying polarization filters, and the subscripts of I correspond to the orientation of the polarizer (0, 45, 90, 135, left circular, and right circular). Thus, the first two components of the Stokes vector (S_0 and S_1) are measured using a linear polarizer orientated at 0° and 90° (horizontal and vertical). The S_0 component is found by summing the two measurements while the S_1 component is determined by subtracting them. Similarly, S_2 is the difference between a measurement with the polarizer at 45° and 135°. I_L and I_R refer to the radiance collected by left and right circular polarizers.

Once the Stokes vector for a ray of light is known, the AoP and DoLP can be readily calculated:

$$AoP = \frac{1}{2}ArcTan\left(\frac{S_2}{S_1}\right)$$
(2)

$$DoLP = \frac{\sqrt{S_1^2 + S_2^2}}{S_0}$$
(3)

AoP ranges from 0 to 180 (not 0 to 360) because the electric field oscillates from -E to E (not 0 to E). DoLP is the ratio of the magnitude of the linearly polarized portion of the light to the total light and ranges from 0 (unpolarized) to 1 (fully polarized).

For modeling purposes, a Mueller matrix is needed. A Mueller matrix is a 4x4 matrix that represents an interaction and, when multiplied to a stokes vector, produces the Stokes vector of the light produced after the interaction. In our case, we will be interested in the Mueller matrix predicted by Raleigh scatter² theory being applied to the Stokes vector of unpolarized sunlight:

$$\begin{pmatrix} (1 + \cos^2\theta) & -\sin^2\theta & 0 & 0\\ -\sin^2\theta & (1 + \cos^2\theta) & 0 & 0\\ 0 & 0 & 2\cos\theta & 0\\ 0 & 0 & 0 & 2\cos\theta \end{pmatrix} \begin{pmatrix} 1\\ 0\\ 0\\ 0\\ 0 \end{pmatrix} = \begin{pmatrix} (1 + \cos^2\theta)\\ -\sin^2\theta\\ 0\\ 0 \end{pmatrix}$$
(4)

where θ is the scatter angle, and for brevity the Mueller matrix and incident stokes vector are normalized to unity. Examining the resulting Stokes vector using equations 2 and 3 reveals that the scattered wave will be polarized perpendicular to the plane of scatter with a degree of polarization dependent on the scatter angle:

$$DoLP = \frac{\sin^2 \theta}{(1 + \cos^2 \theta)}$$
(5)

Thus, any E vector scattered to the observer from a point on the intersection of the scattering field (upper atmosphere) and a plane defined by the observer, the vector to the sun, and an arbitrary point in the sky will be perpendicular to said plane (left image of Figure 1). Rotating the scattering plane about the vector to the sun produces another scatter plane with identical geometry (middle image of Figure 1). This rotation can be repeated any number of times to find the orientation of the E vectors for other points in the sky. Thus, for any single observer location, single scatter Rayleigh theory predicts infinitely many planes sharing the sun and observer upon which each scattered ray has an electric field vector oriented normal to the plane and a DoLP as prescribed in Equation 5.

This creates a beach-ball-type pattern with the lines meeting at the position of the sun and antisun. The right image in Figure 2 simulates a finite sampling of these planes where color is added as a visual aid.



Figure 2: For detail, half of the sky hemisphere is depicted; sunset/sunrise is shown for simplicity, but the geometry is the same regardless of sun position.

The points where the planes intersect (again referencing Equation 5) are unpolarized. These points of no polarization and undefined limit of AoP will be will be referred to as neutral points. This theory leads to the conclusion that there are two neutral points in the sky: on the sun and the anti-sun. As will become evident, however, since light scatters more than once before reaching an observer, this is not the case.

This theory will serve as a baseline in understanding data collected with a sensor developed and built by Polaris Sensor Technologies. Section 2 will discuss the sensor used to collect data, Section 3 will compare the data to the theory, and section 4 will discuss conclusions and suggestions for future work.

2. Sensor Description

The Hyperspectral Imaging Polarimetry Sensor (HIPS), shown in Figure 3, is a full sky imaging spectropolarimeter that measures the absolute spectral radiance and polarization state of natural down-welling sky light over approximately 2π sr⁵. The sensor can semi-continuously collect data at user defined intervals through a custom software provided with the system. The system includes an environmentally isolated VNIR hyperspectral imager and rotating polarizer element mounted on a quickset pan-tilt tripod.



Figure 3: HIPS performing a collection sweep at 56° elevation.

The HIPS sensor collects polarimetric data through a division of time approach, which in this case relies on the continuously rotating polarizer. The polarizer position is indexed and the velocity is set so that the polarizer doesn't rotate more than 22.5 degrees in an integration time. Although the polarizer changes position throughout the integration time, its position is known so a Fourier transform can be performed on the signal to extract the Stokes Vector.

Calibration

A NIST-traceable radiometer attached to an integrating sphere lit with halogen bulbs provided a uniform scene with known radiance for radiometric calibration. A spectrometer was then used as a source to map the spectral lines to the illuminated pixels and derive the relation between pixel location and associated wavelength (figure 4).



Figure 4: Known Spectrum from spectrometer source (bottom) compared to signal from HIPS imager (top)

Finally, a high-extinction polarizer was placed in front of the integrating sphere and HIPS recorded data with the polarizer at multiple orientations to provide a polarimetric calibration. For a given orientation of the calibration polarizer (α), the imager captures images while the internal polarizer is rotating and its orientation (θ) is known throughout the exposure. This produces a cosine squared curve (the signal) convolved with a (known) comb function as described in figure 4. A Fourier transform can be performed on data collected during a full rotation (or multiple rotations) of the polarizer to extract the expected cosine squared curve from Malus' law. A deeper look into the transform reveals a simple extraction of the Stokes vector. Malus' law can be trigonometrically decomposed:

$$I_{inc} \cos^{2}(\theta - \alpha) = \frac{I_{tran}}{2} (1 + \sin(2\alpha)\sin(2\theta) + \cos(2\alpha)\cos(2\theta))$$
(6)



Being the orientation of the calibration polarizer, α is a constant. Sin(2 α) and cos(2 α) are simply Fourier coefficients of the Fourier sine series and Fourier cosine series of the signal; Moreover, cos(2 α) and sin(2 α) are S₁ and S₂, respectively, for linearly polarized light with an AoP of α . After a quick look at A₀ we see

$$\frac{1}{\pi} \int_0^{2\pi} \frac{I_{\text{tran}}}{2} \left(1 + \sin(2\alpha) \sin(2\theta) + \cos(2\alpha) \cos(2\theta) \right) d\theta = I_{\text{tran}} = S_0$$
(7)

Thus, the Stokes vector of the incident light comes directly from a Fourier transform of the measured curve. The derived Stokes vector is then compared to the known Stokes vector of the light from the integrating sphere passing through the calibration polarizer for several orientations to characterize and account for any imperfections in the system.

Operation

HIPS can be run in two different modes: static and kinetic. The static collection mode enables the sensor to stare at a scene and collect data in prescribed intervals. The number of polarizer rotations (frame averages) can also be varied. A kinetic collection has the same functionality as the static collection mode but also utilizes the pan tilt stage to sweep the sky, taking samples at prescribed azimuth intervals. At each azimuth collection point, the GPS time is recorded to file. The user inputs the integration time, elevation angle, azimuth start angle, azimuth step and the number of steps. A scan can be performed up to 360°, or a subset thereof. An option of one or two passes at separate elevation angles can be performed. The sensor has a FOV of 72°, therefore to get a complete hemispherical scan of the sky, a second pass must be performed.

The sensor was designed with a sun avoidance mode to allow data to be collected even when the sun is directly in the FOV, which would otherwise saturate a large portion of the focal plane array. Each time an image is taken, the software analyzes the number of saturated pixels in the scene. If the number of saturated pixels exceeds the user defined threshold, the sun centroid is calculated and a sun blocking flag is moved into position and occludes the sun. By doing so, the data adjacent to the sun is not lost, capturing more of the sky. The sun avoidance can be turned on or off in the software before a kinetic collection is started. Additionally, the user can manually control the position of the sun blocking flag.

3. HIPS Data

The HIPS system was used to collect data in January, 2018 in Huntsville, AL. It was a mostly clear, hazy day with the sun at roughly 25° altitude. Each image features the sun being blocked by the sun flag near the bottom middle of the image, effects of reflected sun light above the blocked sun, the neighboring nine story building in the bottom left, a small cloud in the bottom right and left corners, and slight asymmetric stretching in the azimuthal direction due to the movement of the sun during collection.



Figure 6: HIPS data of the Radiance (top), Degree of Linear Polarization (middle), and Measured Angle of Polarization(bottom) of the sky integrated over 440nm to 450nm. To save space, all subsequent images will use these respective scales.

The DoLP and AoP both exhibit the deviation from the model's expectation caused by multiple scattering. One neutral point is seen above the sun in both images, and another can be seen to be forming under the sun in the AoP image. The radiance is also detectably impacted by multiple scattering; it is not apparent from the panoramic image, but mapping the image onto a hemisphere offers more insight.

The known azimuth and elevation angles of each pixel are used to map each image onto a hemisphere (Figure 7). S_0 is of course highest at the sun and decreases as the scatter angle increases, but another factor is the optical density of the atmosphere. It is thinnest at zenith and thickest at the horizon, and as the atmosphere gets thicker multiple scattering has a larger impact. This, in itself, creates a pattern that is dimmest at the zenith and increases as declination approaches 90°, but the placement of the sun shifts the dim spot. Also, it can now more easily be seen that the DoLP is approximately laid in concentric circles around the neutral point increasing as the scatter angle approaches 90°. Also in the DoLP mapping, the zenith point is marred by a combination of imperfect position mapping and the motion of the sun (and thus sky pattern) during collection.



Figure 7: HIPS Data of S0 (left), DoLP (middle), and AoP (right) Mapped onto Hemispheres. Viewed from Above with Zenith in the Center

Lastly, the AoP appears to have another neutral point (at the zenith) that is not present when examining the DoLP. This is merely an artifact of how the data is collected: the sensor collects data from a single column of points from zenith down to 72° declination, then turns about zenith to collect the next column. This leads to a different "horizontal" axis for each measurement, thus an undefined horizontal axis for zenith (Figure 8).



Figure 8: As HIPS rotates to scan the sky, its internally defined x-axis rotates with it.

The defined horizontal axis has no effect on the value of S_0 nor DoLP, but holds the very meaning of AoP. To achieve an AoP pattern that better matches the theory depicted in Figure 2, a coordinate transform can be performed to give every measurement the same coordinate system; For each orientation of HIPS, the measured E vectors are rotated opposite the rotation executed by HIPS to reach the new measurement position. Rotating the vectors of each orientation results in the AoP of every E vector being determined by the same axis.



Figure 9: HIPS AoP after Rotating Coordinate System. (for this particular data, the y axis was rotated to align with the sun)

This produces a pattern, shown in Figure 9, that is much closer to what the single scatter model predicts (no neutral point on zenith), but the neutral points produced by multiple scattering are still present, as well as the 0/360 disagreement due to the sun's motion during collection.

Rayleigh scatter is strongly dependent upon wavelength; Figure 10 shows the measured radiance averaged over a large section of clear sky vs wavelength. As expected, blue light is scattered much more than any other.



Figure 10: Spectrum of scattered sunlight within boxed region

The DoLP also varies with wavelength, though not as drastically. Within the spectral range of the sensor, the largest difference in DoLP for a given point seems to be around 10% and the peak is closer to 600 nm. Figure 11 shows the relation between DoLP and wavelength for a section of high altitude sky near 90° away from the sun

(theoretically highest DoLP in the sky). There are many atmospheric variables that affect the DoLP, but thus far the data shows that the max tends to be between 550 and 650 nm and spikes in DoLP (the largest visible spike is from O_2 at 760nm) correspond to dips in radiance where the polarization is augmented by molecular scattering (absorption and reemission due to electron excitations).



Figure 11: DoLP Spectrum of the sky

A metric for the amount of polarized light there is in the sky, P, can be constructed by using the unnormalized DoLP (or normalized DoLP multiplied by S_0). That subsequently multiplied by a given sensor's spectral response can dictate the sensor's optimal operating spectral band for polarimetry. Figure 12 shows P for the points in the previously described section of sky multiplied by the spectral response of the detector in SkyPASS.



Figure 12: Spectral polarization signal for SkyPASS

The AoP also seems to have a slight dependence on wavelength. It is easiest to see the differences near the sun because the neutral point appears to move relative to the sun. Figure 13 compares some AoP images near the sun for different wavelengths.



Figure 13: Single AoP image analyzed in four wavelengths: 891nm (top left), 760nm (top right), 500nm (bottom left), 396nm (bottom right).

Noise on the imager (especially in the first and last images) makes the exact location of the neutral point uncertain by visual inspection. However, its location can be inferred by the AoP's divergence. The neutral point rises to its highest point around 500nm and lowers (nearing the sun) as the wavelength departs from the visible band.

4. Summary

Using a sensor and software built and developed in house, Polaris Sensor Technologies has collected and examined spectrally resolved polarimetric data of the scattered sunlight in the sky. This data can be helpful in understanding the sky polarization, demonstrates the capabilities of HIPS, and will contribute in the optimization of SkyPASS. The AoP's dependence on wavelength will be further analyzed and data of other sky conditions will be collected in an effort to better characterize the effects of atmospheric variables so they may be accurately measured and/or predicted.

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